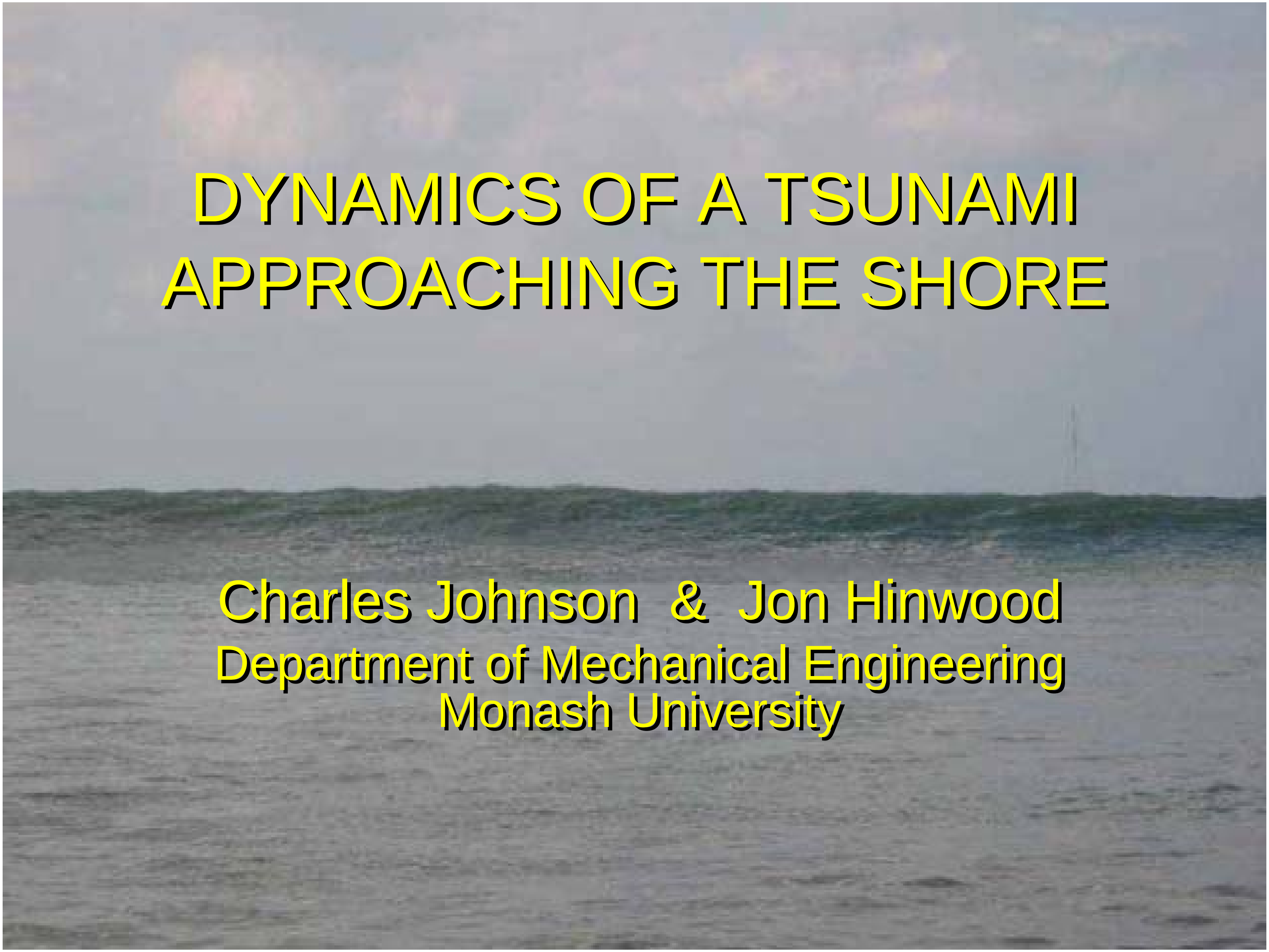

DYNAMICS OF A TSUNAMI APPROACHING THE SHORE

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Uses of predictions of the dynamics of tsunami

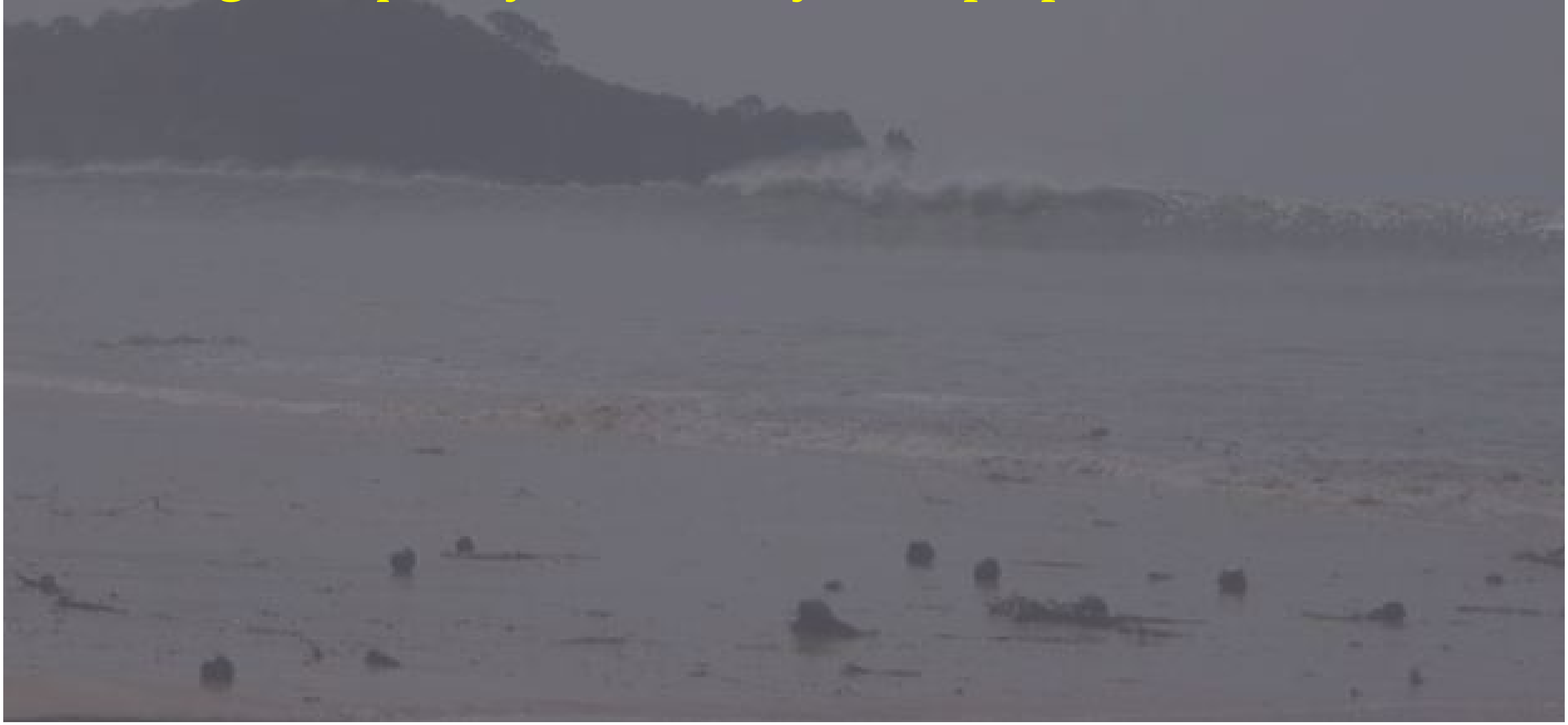
- Escape - time of arrival, height
- Survival and damage -

  number & period of waves

  form of front
- Inundation - depth, duration
- Loads on structures - depth, velocity, form
- Scour of sediments - velocity, duration

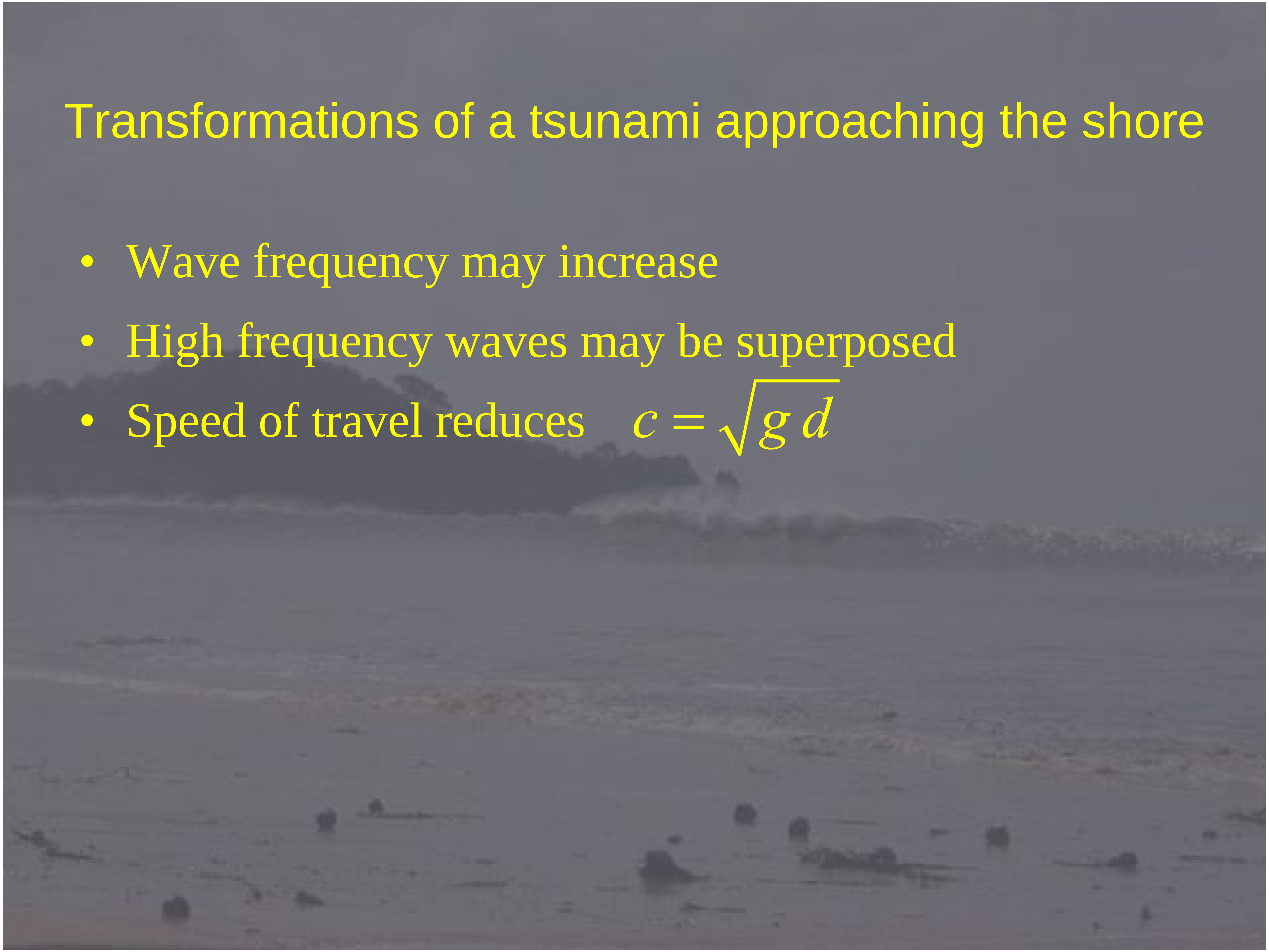
Transformations of a tsunami approaching the shore

- Wave frequency may increase
- High frequency waves may be superposed




Transformations of a tsunami approaching the shore

- Wave frequency may increase
- High frequency waves may be superposed
- Speed of travel reduces $c = \sqrt{g\,d}$



Transformations of a tsunami approaching the shore

- Wave frequency may increase
- High frequency waves may be superposed
- Speed of travel reduces $c = \sqrt{g\,d}$
- Wave height increases

$$\text{Energy flux} = \text{constant} \propto c\,g\,H^2$$

$$H \propto 1 \big/ d^{1/4}$$

Transformations of a tsunami approaching the shore

- Wave frequency may increase
- High frequency waves may be superposed
- Speed of travel reduces
- Wave height increases
- Wave front steepens
- Wave may break

Comparison of dynamic models

Prediction of waves at the shore

- Factors included in equations — Need for factor
- Factors neglected — Consequences
- Simplifications in solutions — Consequences

Method used in this paper

- Factors included in equations — Need for factor
- Factors neglected — Consequences
- Simplifications in solutions — Consequences
- Outcomes

Comparison of dynamic models

Full 3D hydrodynamics

- Factors included – all processes – even if unimportant
- Factors neglected – bathymetry and ocean wave poorly defined – loss of apparent accuracy
- Simplifications – few – computation time large and only particular cases can be simulated

2D hydrodynamics

- Factors included – all processes to some extent – less "wasteful" than 3D models
- Factors neglected – shallow water equations omit vertical accelerations $\rightarrow$ cannot directly model breaking – bathymetry & ocean wave poorly defined …
- Simplifications – depend on model but required for speed

Comparison of dynamic models

1D hydrodynamic-numerical model

- Factors included – may include beach profile, accelerations & friction perpendicular to shore – can model wave transformation, long travel
- Factors neglected – shallow water equations – cannot directly model breaking
- Simplifications – depend on model – many particular cases can be simulated

Shallow water or long wave equations

- Retain the horizontal accelerations
- Omits the vertical acceleration
- Assumes hydrostatic pressure (small surface curvature)
- Similar velocity profiles at all sections
- Cannot directly model steep wave fronts or flow over irregular bed

Comparison of dynamic models

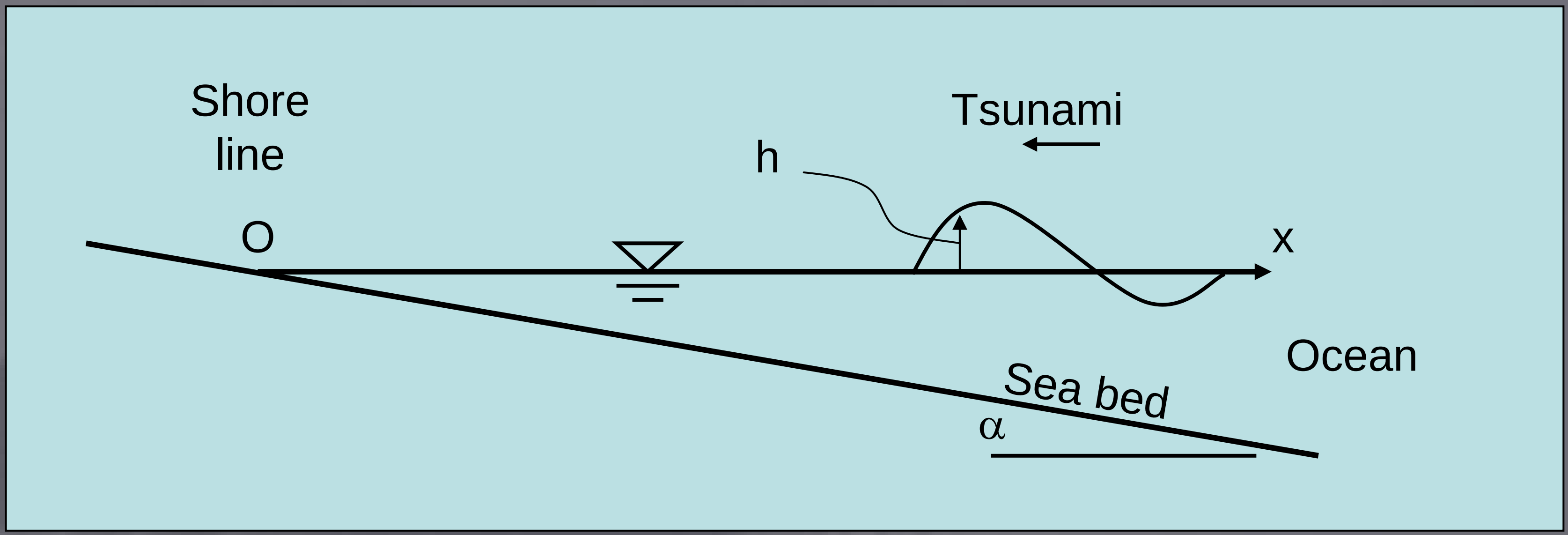
1D hydrodynamic-numerical model

- Factors included – accelerations & friction perpendicular to shore may include beach profile – can model wave transformation, long travel
- Factors neglected – shallow water equations – cannot directly model breaking
- Simplifications – depend on model – many particular cases can be simulated

1D hydrodynamic-analytical model

- Factors included – accelerations perpendicular to shore – wave transformation, moderate travel
- Factors neglected – shallow water equations – breaking …
  – plane beach, no friction – moderate travel
- Simplifications – depend on model – gives families of solutions

Tsunami approaching Shore - 1D Analytical Model



Equations of conservation of mass and momentum

$$\frac{\partial}{\partial t} h(x,t) + \frac{\partial}{\partial x}\{u(x,t)[x \tan\alpha + h(x,t)]\} = 0 \qquad (1)$$

$$\frac{\partial}{\partial t} u(x,t) + u(x,t)\frac{\partial}{\partial x} u(x,t) + g\frac{\partial}{\partial x} h(x,t) = f(u(x,t)). \qquad (2)$$

Method of analysis

First the equations are non-dimensionalised using scale *L*

$x \to x/L \qquad h \to h/\alpha L \qquad u \to u/\sqrt{(g\alpha L)}$

By transforming coordinates,
a pair of linear first-order differential equations is obtained

The transformation is similar to the method of characteristics used in wave motion and flood wave modelling

$$\xi = x + h(x,t), \quad \eta = t - u(x,t) \tag{8}$$

Solution

Solution is in the form of 2 integral equations, one for the velocity

$$u(\xi, \eta) = \frac{1}{\sqrt{\xi}} \int_{\rho=0}^{\infty} \mathbf{J}_1(2\rho\sqrt{\xi})\,\rho\,d\rho \times$$

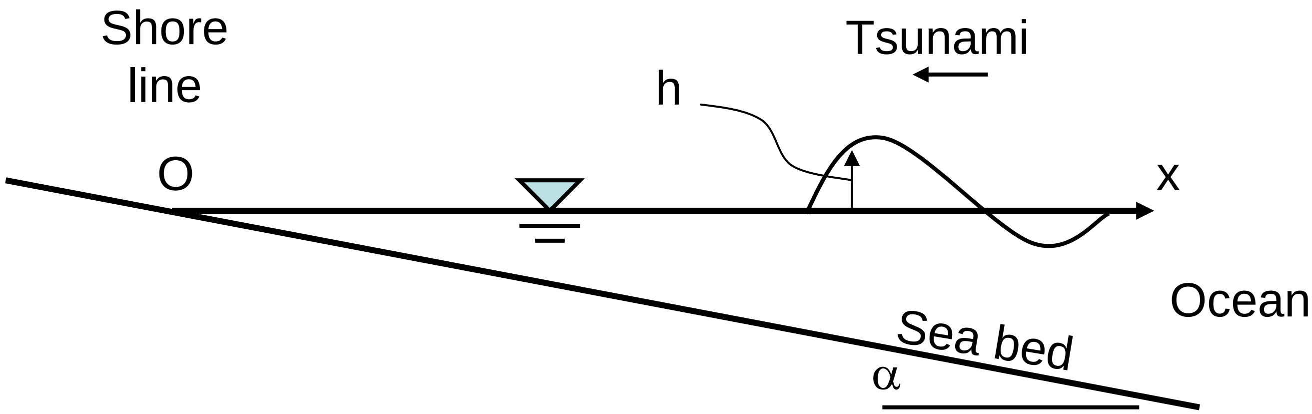
$$\{A(\rho)\cos(\rho\,\eta) - B(\rho)\sin(\rho\,\eta)\},\ (23)$$

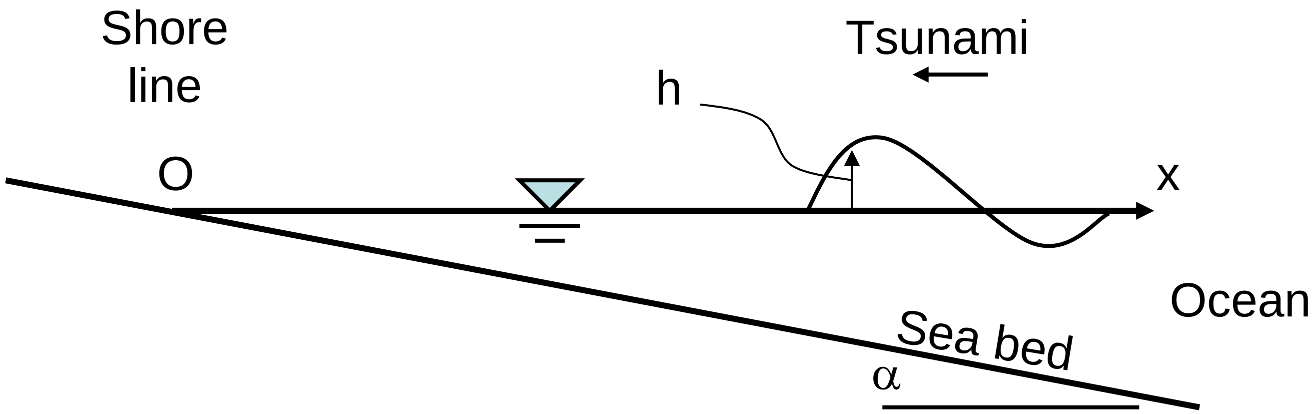
and a similar equation for the energy, from which *h* may be found

Problem remaining is to convert to (*x, t*) coordinates

Conclusions

- 1D analytic modelling is possible
- It can provide families of solutions fast
- It can be used to check numerical models
- Bed friction is difficult to include
- Irregular bathymetry cannot be used


Shore line
Tsunami
h
O
x
Ocean
Sea bed
α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