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MODELS OF OVERLAND FLOW OF A TSUNAMI

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ABSTRACT: The damage and disruption caused by a tsunami depend on its height, speed and the distance the tsunami penetrates inland. While the height, speed of travel, water particle velocity and ultimate distance inland may be estimated using sophisticated numerical modeling, there appears to be a need for simpler and faster estimates, applicable to a range of conditions. Three flows for which there are theoretical or experimental data have been considered as models of the overland flow, they are:

- Shoaling waves and surging breakers
- Tidal bores
- Roll waves

The possibilities of adapting these model flows to the overland flow of a tsunami are examined.

Wave theory and design data have been focused on wind waves but tsunami have a much smaller wave steepness as a consequence of their great wave length, and extrapolation of wind-wave results appears to be quite unreliable. Solitary wave theory provides equations for wave speed and runup which appear reasonable and have been validated for shorter waves. Analytical studies have found that maximum runup depends almost exclusively on the initial wave energy and can be modeled exactly using linear equations. The solitary wave appears to provide the most useful model overall.

Often a breaking tsunami resembles a tidal bore, the equations for which are the well-known hydraulic jump equations. This model is locally valid and can be applied easily to accurately estimate the surge velocity if the surge height is known, but as it does not include bed friction the changing surge heightas the tsunami propagates inland cannot be predicted by this model. This model cannot predict runup. Roll waves are a succession of bores linked by gradually-varied flow, which include bed friction. The roll wave model provides the best representation of the physics of tsunami propagation but the cases studied to date are not applicable to tsunami.

1. INTRODUCTION

The damage and disruption caused by a tsunami depend on its height, speed and the distance the tsunami penetrates inland. While the height, speed of travel and ultimate distance inland may be estimated using sophisticated numerical modelling, such estimates apply only to the very specific conditions of the model run. There appears to be a need for simpler and faster estimates of the upper and lower bounds of these quantities, applicable to a range of conditions.

Simple equations for speed of travel and overland distance travelled have been given by Bryant (2001) who collated both theoretical and experimental results, while some disaster management agencies have in-house guidelines. At present Byant's equations provide as a good an estimate as any other method, but by consideration of model flows it is hoped that the many factors necessarily omitted from such simple guides may be included where they affect accuracy.

Three flows have been identified as models of the overland flow:

- shoaling waves and surging breakers on a beach
- tidal bores
- roll waves.

In the following sections the requirements of a model of overland flow are outlined and then the possibilities of applying these model flows to the overland flow of a tsunami are considered. Some simple guidelines for their use are developed. In the following pages, citations have generally been made to standard treatises and engineering manuals rather than the original scientific papers, as the former are more readily available in the tsunami-prone countries and those sources have usually collated and compared data and theory from several sources.

2. REQUIREMENTS OF AN OVERLAND FLOW MODEL

Near its source and in deep ocean waters a tsunami generally has the form of a train of waves of period in the order of 15 minutes. The leading edge of the waves may be a trough or a crest depending on its generation conditions and location relative to the impact site under consideration. The first wave is often the highest. As the waves run into rapidly shoaling water at the edge of the continental shelf some energy is reflected, prolonging the duration of wave action in the ocean. In addition, the initial waves often become unstable and break into groups of waves of period 4 to 6 minutes. In very shallow water the leading wave front may become unstable and form a group of waves of period 10 to 30 sec superimposed on the front of the longer period tsunami wave.

The long waves are transformed as they move into shallower water since their speed of travel, *c*, depends on the water depth. For a wave of height, *H*, much less than the local water depth, *y*,

$$c = \sqrt{gy} \qquad (1)$$

where g is the acceleration of gravity. From considerations of mass and energy concentration, H must increase as depth decreases. The first order approximation, valid for $H << y$, is

$$H \propto y^{-1/4} \qquad (2)$$

In deep water a tsunami normally has a smooth, continuous profile. In shallow water the variation of the speed with depth results in the wave crest travelling faster than the trough, or leading edge of the wave, increasing the steepness of the wave face. If this process continues long enough the wave forms a surge with a nearly vertical face and usually forms a surging breaker.

As the wave continues to move shorewards, it commences to cross dry land (either land above the undisturbed waterlevel or land left dry by the preceding tsunami wave trough). Almost always this land – the beach – slopes up more steeply than the nearshore sea bed and rises above normal sea level, as shown in Fig. 1. The water forming the wave must therefore rise over a short distance, transforming some of its energy into elevation and effectively losing energy in so doing.

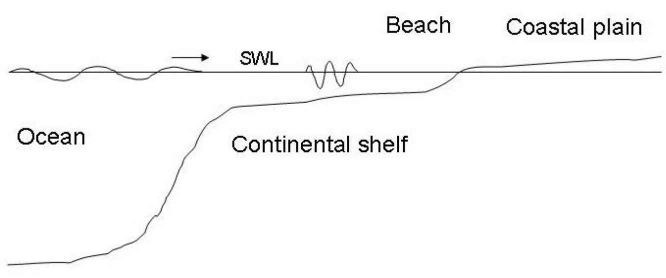

Fig. 1 - Typical depth profile from ocean to shore

Inland of the beach face the land may fall into tidal wetlands, may remain nearly level as a coastal plain, or may rise rapidly as sand dunes or residual land forms which would restrict wave travel. the waves will travel across the land surface, losing energy through friction with the land surface, vegetation and buildings, and by transforming some energy as the land rises inland.

We will consider only the shoreline crossing and the inland flow across dry land, but must be aware of the wave transformations which have occurred as they define the initial form and energy of the wave. For these two stages of the wave propagation the following considerations apply:

Stage 1 – Uprush on the beach face

- Wave moving from travelling in water to crossing dry land
- Land surface sand with no bed forms
- Length of travel short so friction unimportant
- Land slope 0.01 to 0.1
- Beach top elevation above still water level 1.5m
- Vegetation – none (fringing mangroves not considered here)
- Buildings – none on beach, may be at beach top
- Incident wave case considered: H = 3 m (for y = 5 m) and T = 6 min

Stage 2 – Inland flow

- Wave moving from travelling across dry land
- Land surface natural soil or low vegetation
- Extent of coastal plain - unlimited
- Length of travel long, so friction important
- Land slope 0.001 to 0.0001 – small so flow is effectively in steady motion locally
- Vegetation – low growth considered with ground friction or trees of fixed resistance coefficient
- Buildings – none or specified arrangements
- Incident wave – coming from Stage 1 or otherwise based on the same wave

3. SHOALING WAVES AND SURGING BREAKERS

There is considerable theoretical and experimental information on the behaviour of waves propagating normal to a beach. This information has been embodied in design curves and formulae (e.g. CERC, 1984) which are adequate for the engineering design of beach works and coastal structures exposed to wind waves. Wave theories are based on conservation of mass and momentum with most theories neglecting friction or energy dissipation. In experiments on wave shoaling, breaking and runup, friction on the sea bed acts but is important only for second order effects such as the generation of net water and sediment movements. Significant energy dissipation occurs in wave breaking and the energy losses in these highly unsteady flows can only be studied through experiments or in a research context using specialised numerical models. Two steady flows which include breaking waves and for which useful analytical models are available are the hydraulic jump and roll waves.

The information available from wave theory, experiments and design curves relevant to tsunami applies to Stage 1 of their onshore motion and is:

- Conditions for a breaking surge to form, specifically the water depth at breaking and the breaker height as functions of the beach slope and the deep water wave steepness.
- Maximum wave runup as functions of the beach slope and the deep water wave steepness.
- Wave energy reflected.

For waves of very long wavelength, and hence small deep-water steepness, the design curves (CERC, 1984) based on large scale flume experiments show that a surging breaker will form. A surging breaker is very often observed at some stage as a tsunami approaches the shore. The deepwater wave steepness for the case study tsunami is of the order of 10^{-6} which is 2 orders of magnitude smaller than the data, making use of these curves questionable. Further down the scale, the tidal wave has a deep water steepness at least one order of magnitude smaller than a tsunami and generally does not arrive at the shore as a surging breaker (In some localities a surging breaker or tidal bore does form, generally where the tide is propagating into a river or inlet against an outflowing current which effectively shortens the wave length and increases the steepness of the tidal wave).

In view of this difficulty, two broad approaches are possible to utilise existing wave breaking and other data. The first considers one wave within the tsunami equivalent to one wind wave (or laboratory wave) and follows that wave shorewards through shoaling, breaking and runup. The second approach starts with the tsunami already in shallow water, usually but not necessarily broken as a surge, and considers this equivalent to a wave or surge formed from any other wave motion. The first approach has the advantage that the tsunami properties are more likely to be known in the ocean and the surge characteristics may be determined, but there is no simple and reliable method of predicting the transformation as the tsunami shoals. The second approach has the advantage that existing results for surges (and some for other waves) may be directly applied, but presumes that the initial size and speed of the wave in shallow water are known e.g. from observations of tsunami.

For waves of small steepness ($H_0/L_0 << 1$) in shallow water ($L > d/2$) solitary wave theory is recommended (Wiegel, 1964; CERC). Here H_0 and L_0 are the deep water wave height and length respectively, L is the wavelength in water of depth y. At breaking the wave height H_b is predicted by solitary wave theory (Munk, 1949) to be

$$\frac{H_b}{H_0} = \frac{1}{3.3\left(H_0/L_0\right)^{1/3}} \tag{3}$$

Data from Munk (1949) confirm this result down to a deep water steepness of 0.004, as shown in Fig. 2. Trial calculations based on the test case in Section 2 gave an estimate of $H_b \approx 8$ m which is plausible, considering that friction and wave reflection have been neglected and would reduce this value. More recent theoretical studies of the solitary wave have shown that it may break as a spilling, plunging or surging breaker, as do wind waves, but general criteria have not been obtained and the theoretical results have not been experimentally tested.

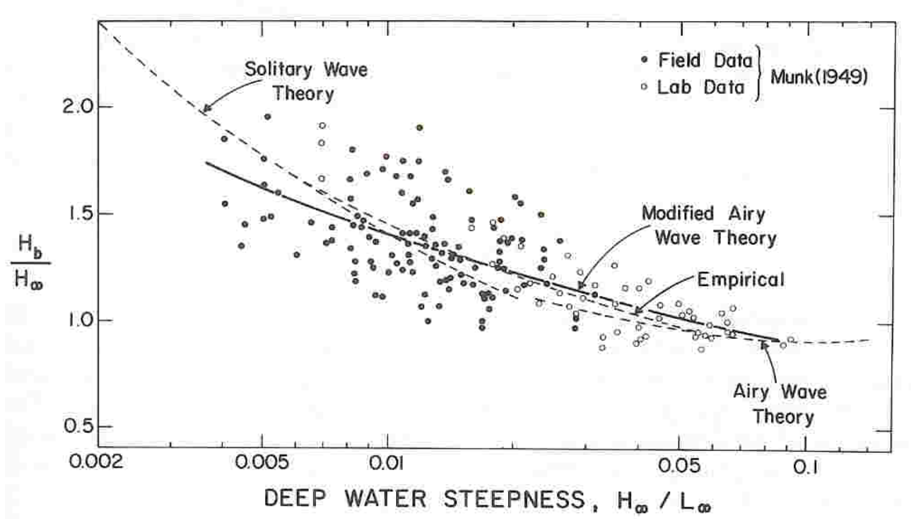

Fig. 2 - Wave breaker height. --- Eqn (3) After Komar (1976)

Based on the equations given by Wiegel, approximations for the velocity of a solitary wave, c, and the maximum speed of the water in it are given by

$$c \approx \sqrt{gd}\left(1+\frac{1}{2}\frac{H}{d}\right) \tag{4}$$

$$u \approx \sqrt{gd}\,\frac{H}{d} \tag{5}$$

where d is the undisturbed water depth. Both expressions are accurate to order $\left(H/d\right)^2$; higher order approximations are given in the literature.

The information on runup is based on experiments on beaches or structures of waves of deep water steepness equal to or steeper than 0.0004 (CERC; Silvester, 1974) and the data depend strongly on the steepness, runup being larger for greater steepness. Thus the experimental data cannot be directly used and Wiegel’s extrapolation to apply it to tsunami is not recommended for use. Wiegel also presents data of Hall and Watts from experiments on solitary waves which do not depend on wave steepness and the power law relationships given by Wiegel can be crudely fitted to the equation

$$R \approx 3.5\,H \tag{6}$$

This approximate result was also found from other data by Silvester – more exactly there are weak dependences on relative depth and beach slope.

A totally different approach was developed in the 1960s and 1970s (Stoker, 1957; Meyer and Taylor, 1972) in which the asymptotic behaviour of the non-linear inviscid equations of long waves running up a plane beach was studied. Meyer and Taylor found that a bore (locally very steep wave front) would form and as the wave neared the shoreline the bore would collapse, converting the potential energy into kinetic energy and creating a rapid uprush across the shoreline. The maximum runup was then given by the initial energy of the bore being converted into potential energy of elevation at the maximum runup. This result meant that the runup depends only on the initial energy of the wave and can be predicted by linear theory. In a series of experimental studies Yeh (Yeh et al, 1989) found that the theory systematically overestimated the velocity of advance by about 20% and hence overestimated the runup, but suggested that some of the difference was an experimental artefact, leaving the issue unresolved.

Prior to these studies, Carrier and Greenspan (1958) had obtained a general analytical solution for the non-linear movement of a solitary wave over a beach but their solution was too cumbersome to be useful. Carrier et al (2003) used a similar analytical approach with a numerical evaluation step. Key findings from 4 sample calculations and some analytical exercises were:

- The maximum runup depends only on the shoreline velocity and hence on the initial energy, and may be predicted exactly by linear theory, as found by Meyer and Taylor. The runup was a maximum for the case where the wave was most like a solitary wave and was $R/H = 2.76$.
- The runup is increased if the first positive wave is preceded by a small negative wave, which reflects steepening the incoming positive wave. A large negative wave also reflects but reduces the incoming positive wave.
- Maximum landwards and seawards velocities occur at the shoreline, i.e. the wave front for the wave traversing initially dry land. For predominantly positive waves the maximum landwards velocity is larger than the seawards and conversely if the wave is predominantly negative. The landwards velocity was a maximum for the case where the wave was most like a solitary wave and was $u_{\max} = 1.63\sqrt{gH}$.

This solution method appears to be the most effective to date and, with some validation, could well be used to generate curves for engineering design.

Some of the incident wave energy will be reflected from the beach, and by changes in bed elevation further offshore. Wave reflection from the beach will create a partial standing wave, increasing waterlevels at the beach face, but will reduce the wave energy propagating inland. Reflection is described in terms of the reflection coefficient $K_r = H_r/H_i$, where H_i is the incident wave height and H_r the reflected wave height. Data collated by Silvester (1974) show that the reflection coefficient increases as the deep-water wave steepness decreases and as the beach slope increases. The values for the smallest wave steepness tested, 0.005, are K_r = 0.15 for a beach slope of 0.1 and K_r = 0.05 for a beach slope of 0.04 Wiegel (1964) reported that negligible wind-wave energy is reflected from a slope less than 0.08.

4. TIDAL BORES

Tidal bores are observed on some rivers subject to high tidal ranges with resulting rapid water level rise. Most of these rivers have strong fresh water outflows, or constricted tidal channels, generating currents which oppose the propagating tide, increasing its steepness. Equations relating the speed of travel to the water depths are well established.

Tidal bores resemble the motion of a tsunami propagating on water, i.e. early in Stage 1 of the motion considered here. They also closely resemble the motion of a tsunami entering an inlet or river, which contribute to Stage 2 but are not considered here as they are generally less important than the overland flows advancing over a much broader front.

Tidal bores have been analysed in terms of the locally equivalent steady flow, which is a hydraulic jump. Recasting the hydraulic jump equations in terms of a tsunami advancing into still water gives the following equation for its speed of advance, *c*:

$$c \quad = \quad \sqrt{g\,y_2}\left[\frac{1}{2}\frac{y_1}{y_2}\left(\frac{y_1}{y_2}+1\right)\right]^{1/2} \qquad (7)$$

where the variables are defined in Fig. 2.

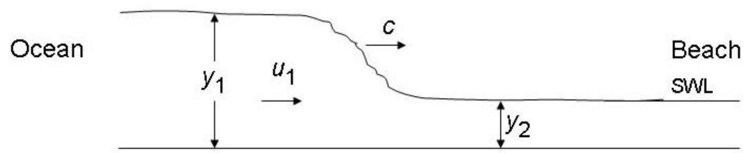

Fig. 3 - Tsunami advancing in still water

The breaker face of a tidal bore is most often a turbulent surging breaker but there are many cases of plunging breakers which provide sport for river surf enthusiasts. A hydraulic jump or bore will have an undular form, rather than a steep face, if the Froude number $F_2 = c/\sqrt{g\, y_2}$ is less than about 1.25 for a smooth seabed. this corresponds to $y_1/y_2 < 1.33$; a larger value applies over a rough bed.

5. ROLL WAVES

The equations describing the hydraulics of open channel flow may be solved for the surface profiles of gradually varied flow – i.e. one dimensional flow in a channel with a given bed slope, S_0, and bed friction, expressed as the friction slope $S_f = f\,u^2/2g\,y$ where f is the Darcy-Weisbach friction factor. In addition to the well known steady flow profiles there are wave train solutions called roll waves. Unlike the waves discussed so far, roll waves do not have a continuous surface profile instead are a series of bores linked by segments satisfying the gradually-varied flow equations in a moving reference frame, as shown in Fig. 4 (Dressler, 1949). The bores each satisfy the tidal bore equations. Continuous, although non-periodic roll waves are predicted by theory and are observed as flow enters a channel, but transform downstream into discontinuous waves. Most analytical and numerical studies have considered only regular periodic waves – essentially a steady-state wave motion.

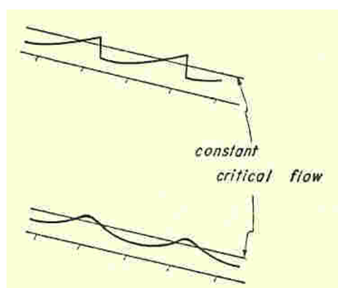

Fig. 4 – Roll wave profiles, upper discontinuous, lower continuous. After Dressler (1949)

Roll waves are a well known problem on dam spillways, where they raise the maximum water level on the spillway. They may also occur in laminar flow (Fig. 5), although a high resistance to flow will suppress them. Regular periodic roll waves may occur provided that

$$2f < S_0 \tag{8a}$$

and

$$F_2 = c/\sqrt{gd} \geq 2 \tag{8b}$$

Criterion (8a) was first derived by Jeffries (1925) to ensure that the energy of infinitessimal waves would increase and was confirmed experimentally by him, while (8b) was obtained by Henderson (1966). They are equivalent as the undisturbed flow is assumed to be a uniform flow ($S_f = S_0$). This criterion is not a sufficient condition for roll waves to occur. It is clear that roll waves dissipate energy and hence regular periodic roll waves cannot exist on an adverse slope ($S_0 < 0$) – a conclusion confirmed by criterion (8).

From experiments in a long but very shallow channel, Brock (1969) showed that roll wave height and celerity increase with distance downstream from a quiet inlet, even for non-dimensional distances x' ~ 104. Here u_0 = velocity of uniform flow in channel of slope S_0
d_0 = depth of uniform flow
$L_0 = d_0/S_0$
$x' = x/L_0$

Fig. 5 – Roll waves on laminar flow – fountain at Chadstone Shopping Centre, Melbourne

Ponce and Maisner (1993) showed that roll wave growth was most rapid for $L/L_0 \approx 30$ and showed that waves of this length were dominant in Brock's data, while Balmforth and Mandre (2004) mapped the stability regions and boundaries as a function of domain size and initial disturbance and provided equations for growth rate.

Brock, Ponce and Maisner, and Balmforth and Mandre are among the very few studies to have considered irregular roll waves but, as with all studies of regular roll waves, they considered only positive slopes ($S_0 > 0$). The roll wave model combines surges with unsteady frictional flow and thus captures the physics of the tsunami but up to now there are no solutions or data applicable to tsunami.

6. DISCUSSION

In flowing uphill all flows convert energy to elevation and all flows lose energy through bed friction. Breaking waves and bores also lose energy rapidly through turbulence production. Consequently no flow can propagate unchanged without a source of energy. Models of flow which neglect energy losses can only describe these flows for a limited part of their travel and need to be reapplied at points along the wave's travel allowing for energy loss, which must be computed using other models. The capability of the frictionless models may still be useful, and the simplicity resulting from fewer processes in the model may easily compensate for loss of generality.

The tidal bore does not neglect the energy loss at the bore, and indeed is used to calculate this loss, hence it has a useful role. However the simple tidal bore model does not include bed friction or bed slope and cannot by itself describe the travel and transformation of a tsunami over large distances.

Almost all wave theories neglect bed friction, but some include bed slope. Wind wave data include the effects of bed friction but do not extend to the low wave steepness of tsunami, limiting the applicability of the data and limiting its use in the validation of wave theories. Bed friction may usually be neglected prior to the tsunami reaching the beach, and for waves on the beach face frictional energy loss is small compared with the changes in energy occurring there as the width of the beach is relatively small keeping the friction losses small also.. Hence wave theory has the potential to describe the offshore and nearshore wave transformations, including the initiation of breaking and motion on the beach face. Solitary wave theory treats waves of very low wave steepness and appears to be the best of the wave models.

A quasi-steady flow, permitting the use of the above models, is possible if energy losses are balanced by an energy source. The only source of energy for a tsunami front advancing across level ground or uphill is from the following wave - it has been shown above that the wave front will steepen as the following part of the wave travels faster in the deeper water behind the front. Thus a breaking front may be sustained by a wave which continues to rise behind the front. Clearly this action can only be maintained for a limited time and does not appear to offer practical solutions.

Behind the wave front, accelerations are smaller and bed friction becomes more important. The proportions of the test case lead to both the local and advective accelerations and the friction force per unit mass being of order 0.05 m/s^2, necessitating retention of all three terms. If the effect of bed slope is included, the result is that an unsteady gradually-varied open channel flow model is required in general. Specific cases in which the unsteady term is much smaller lead to the well-known and relatively simple gradually-varied flow equations. A model worthy of consideration, but not considered here, is the dam-burst outflow. In its simplest form the dam-burst outflow is frictionless unsteady flow across a horizontal bed but bed friction has been included in many studies.

7. CONCLUSIONS

Three flows for which there are theoretical or experimental data have been considered as models of the overland flow, they are:

- Shoaling waves and surging breakers
- Tidal bores
- Roll waves

Wave theory and design data have been focused on wind waves but tsunami have a much smaller wave steepness as a consequence of their great wave length. A rough guide to maximum runup is $R \approx 3.5\,H$, although extrapolation of wind-wave results appears to be quite unreliable. Solitary wave theory provides equations for wave speed and runup which appear reasonable and have been validated for shorter waves. Analytical studies have found that maximum runup depends almost exclusively on the initial wave energy and can be modeled exactly using linear equations. Some interesting results from examples evaluated numerically are given. Few of these studies have included bed friction (resistance to flow) and hence are limited to nearshore flow and the flow up the beach face.

Often a breaking tsunami resembles a tidal bore, the equations for which are the well-known hydraulic jump equations. This model is locally valid and can be applied easily to accurately estimate the surge velocity if the surge height is known, but as it does not include bed friction the changing surge height cannot be predicted by this model. This model cannot predict runup.

Roll waves are a succession of bores linked by gradually-varied flow, which included bed friction. The roll wave model provides the best representation of the physics of tsunami propagation but the cases studied to date are not applicable to tsunami.

The solitary wave appears to provide the most useful model overall. It is recommended that the most promising solution method (Carrier et al, 2003) be used to generate design curves and that they be validated experimentally.

8. REFERENCES

1. Balmforth, N.J. and Mandre, S. (2004) Dynamics of roll waves, J. Fluid Mechanics, 514: 1-33.
2. Brock, R.R. (1969) Development of roll-wave trains in open channels, Proc. ASCE, 95 (HY4): 1410-27.
3. Bryant, E. (2001) Tsunami: The Underrated Hazard, Cambridge University press, Cambridge, 320pp.
4. Carrier, G.F., Wu, T.T. and Yeh, H.H. (2003) Tsunami run-up and draw-down on a plane beach, J. Fluid Mechanics, 475:79-99.
5. Carrier, G.F. and Greenspan, H.P. (1958) Water waves of finite amplitude on a sloping beach, J. Fluid Mechanics, 4:97-109.
6. Dressler, R.F. (1949) Mathematical solution of the problem of roll-waves in inclined open channels, Communications on Pure and Applied Mathematics, 2(2): 149-194

7. CERC (1984) Shore Protection Manual, Coastal Engineering Research Center, US Army Corps of Engineers, Vicksburg, Mississippi, USA. 2 vols.
8. Henderson, F.M. (1966) Open Channel Flow, Macmillan,NY, 522pp.
9. Jeffries, H. (1925) The flow of water in an inclined channel of rectangular section, Philosophical Magasine, Ser. 6, 49 (293): 793-807.
10. Komar, P.D. (1976) Beach Processes and Sedimentation, Prentice-Hall, Englewood Cliffs, NJ, USA.
11. Lamb, H. (1938) Hydrodynamics, 6th ed. Dover, NY, USA, 738pp.
12. Meyer, R.E. and Taylor, A.D. (1972) Run-up on beaches, in Waves on Beaches and Resulting Sediment Transport, R.E. Meyer ed., Academic, NY, pp 357-411.
13. Munk, W.H. (1949) The solitary wave theory and its application to surf problems, New York Academy of Science Annals, 51:376-424.
14. Ponce, V.M. and Maisner, M.P. (1993) Verification of theory of roll-wave formation, J. Hydraulic Engineering, 119 (6): 768-773.
15. Silvester, R. (1974) Coastal Engineering, Elsevier, Amsterdam, Netherlands, 2 vols.
16. Stoker, J.J. (1957) Water Waves, Interscience, NY, USA, 567pp.
17. Wiegel, R.L. (1964) Oceanographical Engineering, Prentice-Hall, Englewood Cliffs, NJ, USA, 532pp.
18. Yeh, H.H., Ghazali, A. and Marton, I. (1989) Experimental study of bore runup, J. Fluid mechanics, 206:653-578.f